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Combined forced and natural convection with low-speed air flow over horizontal cylinders

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This article describes experimental work on the mixed convection régime with flow normal to electrically heated cylinders. The forcing velocities used were in the range 0·0085–3 ft./sec (i.e. $10^{-2} < Re_f < 45$) and temperature differences in the range 30 °C to 200 °C (i.e. $10^{-3} < Ra < 10$) were covered.

Correlations are proposed for the forced convection and natural convection conditions. A correlation is also developed for the combined forced and natural convection region by a vectorial addition of the flow parameters, which gives good agreement with the experiments except over a limited range in the contra-flow régime.

1. Introduction

The experimental research to be described in this work was initiated by a requirement to measure the low-speed air velocities which occur in natural convection. Hot-wire anemometers are normally used in situations where the air velocity is large enough for the heat-transfer process to be considered as one of forced convection alone. However, it is important and interesting to ask the question 'How low may the forcing velocity be before the errors caused by the natural convection of the heated wire itself become significant?'

An examination of the previous work which was available showed that this question could not be answered. In fact no significant amount of work on the question could be discovered.

The region in which both natural and forced convection are of comparable significance is extremely complex. The natural convection will cause the fluid to move vertically upwards whereas the forcing velocity may be in any direction. Hence, for example, one would expect a very different heat-transfer behaviour from a heated wire with an upwards forced flow than from a downwards flow.

The programme of work to be described has three objectives, namely (*a*) to discover if the behaviour of heated cylinders in the region of very low air speeds was sufficiently regular and repeatable to permit the construction of a low-speed hot-wire anemometer; (*b*) if this objective was found to be impossible, to discover the limits of normal hot-wire anemometry within which it is possible to operate with confidence before the natural convection of the heated wire causes uncertain performance; (*c*) to add to the store of basic knowledge of heat transfer in the forced, natural and combined régimes with air flows normal to heated cylinders.

2. Survey of previous analytical and experimental work

Several workers have sought an analytical solution to the problem of heat transfer from a cylinder. A number of theories have been put forward for the case of either natural or forced convection when treated separately but no complete analysis is available for either situation. The theories have given an insight into the problem and allowed empirical equations to be formed by using the same non-dimensional parameters as appear in the attempted analytical solutions. The case for the use of these non-dimensional parameters is further strengthened by dimensional analysis.

2.1. *Natural and forced convection*

A brief résumé of the available empirical equations for situations where only one régime is predominant is given in table 1, where Re is the Reynolds number $\bar{u}d/\nu$, Nu is the Nusselt number $h.d/k$, Ra is the Rayleigh number $Gr.Pr$, where $Gr = \beta g d^3 \Delta T/\nu^3$ is the Grashof number, and $Pr = c_P\mu/k$ is the Prandtl number. $\bar{u}$ is the mean velocity, d the diameter, ν the kinematic viscosity, μ the dynamic viscosity, h the convective heat-transfer coefficient, k the thermal conductivity,

Author	Equation	Range			
	NATURAL				
van der Hegge Zijnen (1956)	$Nu = 0{\cdot}35 + 0{\cdot}25Ra^{0{\cdot}125} + 0{\cdot}45Ra^{0{\cdot}25}$	$10^{-7} < Ra < 10^9$			
Mikheyev (1956)	$Nu = ARa^n$	Ra	A		n
		10^{-3}–10^2	1·18		0·125
		10^2–10^8	0·54		0·25
		10^8–10^{13}	0·135		0·33
Tsubouchi & Masuda (1966*a*)	$Nu = 0{\cdot}36 + 0{\cdot}52Ra^{0{\cdot}25}$	$10^{-6} < Ra < 10$			
	$Nu = 0{\cdot}36 + 0{\cdot}048Ra^{0{\cdot}125} + 0{\cdot}52Ra^{0{\cdot}25}$	$10^{-6} < Ra < 10^9$			
	FORCED				
McAdams (1954)	$Nu = 0{\cdot}32 + 0{\cdot}43Re_{Mc}^{0{\cdot}52}$	Where $Re_{Mc} = (\rho_\infty \bar{u} d)/\mu_f$			
Hilpert (1933)	$Nu = c\left[Re_f\left(\frac{T_w}{T_\infty}\right)^{0{\cdot}25}\right]^n$	Re_f	c		n
		1–4	0·891		0·33
		4–40	0·821		0·385
		40–4000	0·615		0·466
Collis & Williams (1959)	$Nu = (A + BRe_f^n)\left(\frac{T_f}{T_\infty}\right)^{0{\cdot}17}$	Re_f	A	B	n
		0·02–44	0·24	0·56	0·45
		44–140	0	0·48	0·51
van der Hegge Zijnen (1956)	$Nu = 0{\cdot}35 + 0{\cdot}5Re_f^{0{\cdot}5} + 0{\cdot}001\,Re_f$	$10^{-1} < Re_f < 10^5$			
Tsubouchi & Masuda (1966*b*)	$Nu = 0{\cdot}36 + 0{\cdot}48Re_f^{0{\cdot}5}$	$0{\cdot}5 < Re_f < 10^3$			

TABLE 1. Heat-transfer correlations for natural and forced convection from horizontal cylinders

T the temperature and β is the coefficient of volumetric expansion. Subscripts w, ∞ and f imply wall condition, free-stream condition and film condition, where $T_f = \frac{1}{2}(T_w + T_\infty)$.

2.2 *Mixed convection*

Whilst theoretical solutions have been carried out for the vertical flat plate in the mixed convection régime it has not been possible to do this for the cylinder due to the complexity of the problem. It can be shown by dimensional analysis that

$$Nu = f(Re, T/T_\infty, Ra). \tag{1}$$

Acrivos (1958) showed that the parameter (Ra/Re^2) is of fundamental importance in the study of mixed convection. Introducing this term into (1) in place of Ra gives

$$Nu = f(Re, T/T_\infty, Ra/Re^2). \tag{2}$$

Klyachko (1963) analyzed Yuge's (1960) work and suggested the following equations for spheres

$$Nu = Nu_{\mathrm{fo}}\left[1 + C\left(\frac{Re+D}{Re}\right)\left(\frac{Gr}{Re^2}\right)^{0\cdot25}\right] \tag{3}$$

and

$$Nu = Nu_{\mathrm{free}}\left[1 + \left(\frac{Re^2}{Gr}\right)^{0\cdot2}\right], \tag{4}$$

where C and D are constants and subscript free means the natural convection condition and subscript fo the forced condition.

The particular equation to be used was determined by applying the criterion: (*a*) if the forcing flow was predominant, equation (3) was used and (*b*) if the natural convection was greater, equation (4) was used.

Van der Hegge Zijnen (1956) carried out mixed flow experiments on cylinders and correlated the data by the vectorial addition of the free and forced convective heat-transfer parameters as follows

$$Nu = (Nu_{\mathrm{fo}}^2 + Nu_{\mathrm{free}}^2)^{0\cdot5}. \tag{5}$$

The experiments were carried out for cross-flow but the agreement between equation (5) and the experimental data was unsatisfactory.

Börner (1965) carried out experiments on heated cylinders, spheres and flat plates in both parallel and contra flow. The data were correlated by adding vectorially the Reynolds number of the forcing flow and the term $\sqrt{(\frac{1}{2}Gr)}$, which has the form of a Reynolds number. The results correlated within $\pm$ 20 % but were not in a form which enable comparison to be made with the present work.

3. Apparatus and experimental techniques

3.1. *Description of test rig*

The apparatus consisted of an electrically heated horizontal cylinder which was subjected to a range of low-speed air flows emanating from a nozzle connected to the outlet of a settling chamber. This arrangement was protected from draught by a perspex enclosure, the sides of which were 10 in. from the test specimen and nozzle. Nylon gauze outlets were provided opposite the air jet and in the roof of the enclosure. The base of the enclosure was mounted on a swivel structure so that the three forcing flow orientations of vertically upward, downward and horizontal could be easily obtained.

The cylinders used were all 4·75 in. long and consisted of stainless-steel hypodermic tubing of diameter 0·0495 and 0·032 in. and platinum wire 0·004 in. in diameter. It was essential that the length and diameter of the test pieces were known precisely, this being of particular importance in the Rayleigh-number calculation. Shadowgraph techniques were used for measuring both tube and wire diameters.

The cylinders were heated by means of a d.c. power supply provided by two 12 volt heavy-duty batteries connected in parallel for the stainless-steel tubes and in series for the wire. The voltage drop over the central 2 in. of the hypodermic tubing was measured by a digital voltmeter and the current by a standard resistance in series. A similar system was used for the wire over its whole length.

To produce a two-dimensional heat-transfer situation it was essential that the longitudinal temperature variation along the test cylinders be kept to a minimum. Allowance was in fact made for end-conduction effects and the accompanying longitudinal temperature variation, in Nusselt-number calculations.

The surface temperature of the hollow cylinder was obtained by inserting a 42 s.w.g. chromel alumel thermocouple into each end of the cylinder. When equilibrium was established these thermocouples gave the inside surface temperature. The e.m.f. was recorded by a digital voltmeter reading to 0·5 % or a potentiometer reading to a much higher degree of precision. The wire temperature was measured by its resistance change. A calibration was carried out using an oil bath over a range 20–300 °C and the variation was found to be linear.

A range of low-speed air flows was provided by a gasometer and fan. The gasometer had a capacity of 10 cu.ft. and the useful time of descent lay between 5 min and 3 h. For higher flow rates a centrifugal fan was used, the flow from the fan being measured by a calibrated orifice. Each joint in the pipe assembly was carefully sealed to eliminate any possibility of leakage and the obvious resulting errors, particularly at low air-flow rates.

The settling chamber was 6 in. square in cross-section and 12 in. in length. The nozzle exit was 6 in. wide by 2 in. high, the convergence being effected by two circular arcs. The air was fed into the settling chamber area through a short piece of blanked-off pipe which contained holes drilled both in the blanking end and around the periphery. Two fine mesh screens were incorporated in the settling chamber.

The distribution of velocity over the nozzle outlet was checked with a Disa hot-wire anemometer at a mean speed of approximately 1 ft./sec. Whilst this instrument would not indicate the absolute velocity with any precision it did indicate deviations from the mean which were ± 2 % at most. However the profile of velocity was flat over a large central area and the deviations were greatest near the plane edges of the nozzle. The error due to this slight maldistribution was negligible. It was assumed that no accentuation of this effect would occur at lower speeds.

3.2. *Sources of error*

(*a*) *Variation in rate of descent of gasometer.* The gasholder was used over the central 7 cu.ft. of its range of 10 cu.ft. It was noted that, for slow descent, the speed was not constant but varied from the 7 cu.ft. mean by $+3\,\%$ to $-1\,\%$. This was accounted for when several tests were done for one gasholder charge. The useful range of flow rates over which thermal equilibrium of the cylinders could be achieved was from 2–90 ft.^{3}/h.

(*b*) *Variation of tube and wire diameters.* The diameters of the test cylinders were found to vary by about $\pm 0{\cdot}5\,\%$ (tube) and $\pm 2\,\%$ (wire). Mean values were used as determined by shadowgraph.

(*c*) *Orifice plate calibration.* For flow rates above the useful range of the gasometer the orifice was used, but it was found that the lower range of operation was not covered by B.S. 1042. It was possible to cover a range up to 800 ft.^{3}/h from the gasometer and the orifice was therefore calibrated using the gasometer up to this value. In fact this calibration confirmed the B.S. coefficients.

(*d*) *Temperature corrections.* (i) The internal temperature of the tube wall at the centre position was measured and the outside temperature was inferred theoretically. Calculations at the maximum temperature difference of 200 °C showed this correction to be $0{\cdot}08\,\%$ which was considered negligible. (ii) End-conduction. For the tubes, the effect of end-conduction was allowed for analytically in the programme which calculated the dimensional parameters. For the wire the effect of end-conduction, which caused non-uniformity of resistance, was also allowed for theoretically.

3.3 *Range of tests covered*

The experimental work consisted of a series of tests on the tubes and wire in which the velocity of the forcing air was kept as constant as possible whilst the temperature difference between the cylinder and the air was varied between 30 and 200 °C. This was done for each size of cylinder, each of the three directions of flow and each particular flow rate. The flow rate was adjusted so that the velocity over the cylinder varied between 0·0085 and 3 ft./sec, and this covered the full range up to a Reynolds number of 40, which was considered to be a suitable maximum. Collis & Williams (1959) reported that a change in the form of the relationship for Nusselt number takes place at $Re = 44$.

Consideration of the possible errors gave maximum uncertainties in the values of Nu, Re and Ra of $\pm 2\,\%$ for the cylinders and $\pm 5\,\%$ for the wire, the latter being mainly due to the problem of resistance measurement.

4. Data reduction

4.1. *Evaluation of fluid properties*

Several reference temperatures have been used in the past for the determination of the fluid properties: (*a*) the free stream temperature, T_∞; (*b*) the film temperature, T_f, where $T_f = \frac{1}{2}(T_w + T_\infty)$; (*c*) the wall temperature, T_w. In addition, Sparrow & Gregg (1958) have suggested a further reference temperature T_r, where $T_r = T_w - 0{\cdot}38(T_w - T_\infty)$. The use of this reference temperature is an attempt to eliminate the temperature loading factor, T/T_∞; however, it has not been widely accepted and generally (*a*) or (*b*) are used. In this work all the fluid properties have been evaluated at the film condition. It should be noted that it was also necessary to evaluate Re_∞ in order to evaluate the temperature loading factor index.

4.2. *Determination of non-dimensional parameters*

A computer programme was developed to calculate the Nusselt number, Reynolds number, Rayleigh number and temperature loading factor. This programme made allowance for the end-conduction losses from the cylinder and for the radiation loss. The latter was calculated using values of emissivity for the tubes of 0·15 and for the wire of 0·095 which were obtained from property tables. The maximum radiation loss was 3 % of the total and the maximum correction to the Nusselt number, made by taking into account the end-conduction losses, was also 3 %.

5. Methods of correlation

In order to appreciate the vectorial addition method suggested, it is advantageous to consider a simple approach to natural convection. If the work done by the buoyancy force on a fluid element is equated to the gain of kinetic energy then it may be shown that

$$Re = (2Gr)^{0\cdot5}.$$

If this is true, it should be possible to use the same form of equation for both natural and forced convection, using Re in the forced and $(2Gr)^{0\cdot5}$ in the natural case. It will be shown that a modified form of this substitution is more convenient. Once this substitution has been established from the forced and natural convection equations it is possible to add vectorially the forcing and natural Reynolds numbers to give an effective Reynolds number Re_{eff}.

By adding vectorially and using the substitution of

$$Re = (2Gr)^{0\cdot5},$$

it can be shown that

$$Re_{\text{eff}}^2 = Re_f^2\left[1+3\cdot4\left(\frac{Ra^{0\cdot5}}{Re_f}\right)\cos\theta+2\cdot85\left(\frac{Ra}{Re_f^2}\right)\right], \tag{6}$$

where θ is the angle from the vertically upward direction of the forcing flow.

The natural convection Reynolds number is based on some characteristic velocity in the fluid whilst the forcing Reynolds number is based on the free stream velocity. It is therefore evident that the proposed relationship for a natural convection Reynolds number, i.e. $Re = (2Gr)^{0\cdot5}$, will have to be modified. This will be shown below by comparing the equations for forced and natural convection assuming the same Nusselt number is produced.

For forced convection an equation of the form

$$Nu\left[\frac{T_f}{T_\infty}\right]^{-m} = A + B\,Re_f^c = f(Re_f) \quad \text{(say)} \tag{7}$$

will be used, where A, B, c and m are constants to be found from the experimental data.

First consider the solution for finding m. The data recorded was computed in such a way that the following information was available, Nu, Re_f, Re_∞, Ra, T_f/T_∞, T_∞. From (7) it can be seen that if Re_f is constant for a series of values of Nu and T_f/T_∞ it is possible to determine m with

$$\log Nu = \log f(Re_f) + m \log (T_f/T_\infty). \tag{8}$$

The data was collected and calculated so that for each value of Re_∞ a series of values of T_f/T_∞, Nu and T_∞ were covered. This enabled a polynomial curve fit to be carried out at each value of Re_∞ for Nu *vs.* T_f/T_∞ and T_∞ *vs.* T_f/T_∞. From these it was possible to calculate values of Nu and T_∞ for selected values of T_f/T_∞. By using the relationship $Re_f = Re_\infty \nu_\infty/\nu_f$, Re_f was calculated for each of these values of T_f/T_∞. For each constant value of T_f/T_∞ a polynomial curve fit was carried out of Nu *vs.* Re_f. For selected values of Re_f, Nu was calculated with T_f/T_∞ known. It was now possible to carry out a curve fit of $\log Nu$ *vs.* $\log T_f/T_\infty$ for each value of Re_f. The coefficient of this curve produces a value of m. This procedure was carried out for each value of Re_f and the mean value of m obtained.

With m determined, the problem is now to establish A, B and c. Re-writing (7) in the form

$$X = A + BY^c \tag{9}$$

then introducing an error term α, squaring the equation and summing all values of X and Y yields

$$\sum_{i=1}^{i=n} \alpha^2 = nA^2 + 2AB \sum_{i=1}^{i=n} Y_i^c - 2A \sum_{i=1}^{i=n} X_i - 2B \sum_{i=1}^{i=n} (X_i Y_i^c) + B^2 \sum_{i=1}^{i=n} Y_i^{2c} + \sum_{i=1}^{i=n} X_i^2. \tag{10}$$

For minimum error therefore

$$\frac{d \sum_{i=1}^{i=n} \alpha^2}{dA} = 0, \quad \frac{d \sum_{i=1}^{i=n} \alpha^2}{dB} = 0, \quad \frac{d \sum_{i=1}^{i=n} \alpha^2}{dc} = 0.$$

This then gives three equations with three unknowns, assuming X_i and Y_i are known, from which we can obtain A and B in terms of c, X_i and Y_i.

By substituting for A and B in the third of these equations an equation can be obtained which is a function of c alone, i.e. $f(c)$ and it is possible to find c by using the Newton–Raphson iteration;

$$c = c - \frac{f(c)}{f'(c)}.$$

The initial value for c has to be postulated and when c has been found then A and B can be determined.

Previous workers have used many types of correlation for natural convection but a more general equation is given by

$$Nu = A + B Ra^n. \tag{11}$$

In this work, however, an equation of the following form is suggested

$$Nu[T_f/T_\infty]^{-m} = A + B Ra^n. \tag{12}$$

The temperature loading factor index m used was that obtained from the forced convection tests. The results for A, B and n found from the computer programme did not produce suitable values, as A should have the same value for both forced and natural convection if the vectorial addition method is to produce a fairly simple final expression. The relationship was therefore modified by plotting $Nu\,[T_f/T_\infty]^{-m} - A_{(\text{forced})}$ against Ra and obtaining the index n and the constant B. This modified correlation will be compared with the original correlation later in the text.

6. Discussion of results

6.1. *Forced convection*

All the experimental points have been fitted to an equation of the form used by Collis & Williams (1959), i.e. equation (7).

The temperature loading factor index m was found for all the forced convection data combined to be

$$m = 0{\cdot}154.$$

The value suggested by Collis & Williams (1959) is 0·17 and the value suggested by Hilpert (1933) was 0·096, but this value referred to T_w/T_∞. When converted to apply to T_f/T_∞ it is approximately 0·174. The value of m is dependent upon the values used for the fluid properties and a slight difference in these will produce a change in m.

When the constants A, B and c were calculated for each direction individually, and also for all three directions combined, the values found were as follows:

	A	B	c
Parallel flow	0·293	0·568	0·409
Cross flow	0·525	0·406	0·494
Contra flow	0·278	0·667	0·406
All directions	0·384	0·581	0·439

With a relationship of the form of (7) it is possible for there to be a large difference between the coefficients with little physical change in the shape of the curve. The 'all-directions' coefficients were used for the forced convection correlation and this line has been drawn on figure 1 to compare the correlation with the experimental data for forced upwards convection. It will be observed that the majority of the points lie on this curve. Similar results were obtained for the other two flow configurations. Collis & Williams (1959) used the same type of correlation and, as indicated in table 1, gave the following coefficients

$$A = 0{\cdot}24; \quad B = 0{\cdot}56, \quad c = 0{\cdot}45.$$

These coefficients lie reasonably well within the scatter of those found in the present work. The proposed correlation is shown in figure 2, where it is compared with those referred to in table 1. Between the range of $5 < Re_f < 40$ the correlation compares well with that of other workers, but below $Re_f = 5$ it predicts a higher Nusselt number. The maximum range of deviation of all these correlations is about 10 % at $Re = 5{\cdot}0$ and the proposed correlation lies nearest to that of Hilpert (1933).

The derived correlation for forced convection is therefore

$$Nu\,[T_f/T_\infty]^{-0\cdot154} = 0\cdot384 + 0\cdot581 Re_f^{0\cdot439}. \tag{13}$$

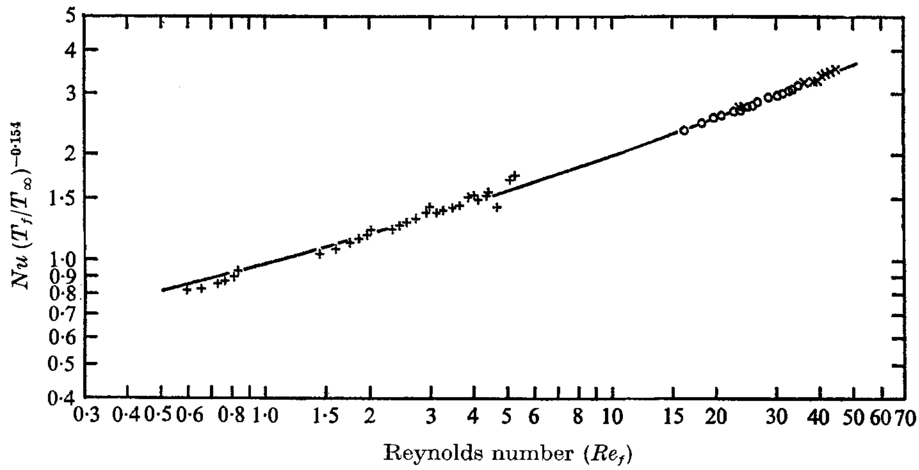

FIGURE 1. Proposed forced convection correlation for parallel flow. Correlation:
$$Nu\,(T_f/T_\infty)^{-0\cdot154} = 0\cdot384 + 0\cdot581\, Re_f^{0\cdot439}.$$
+, d = 0·004 in.; ○, d = 0·033 in.; ×, d = 0·0495 in.

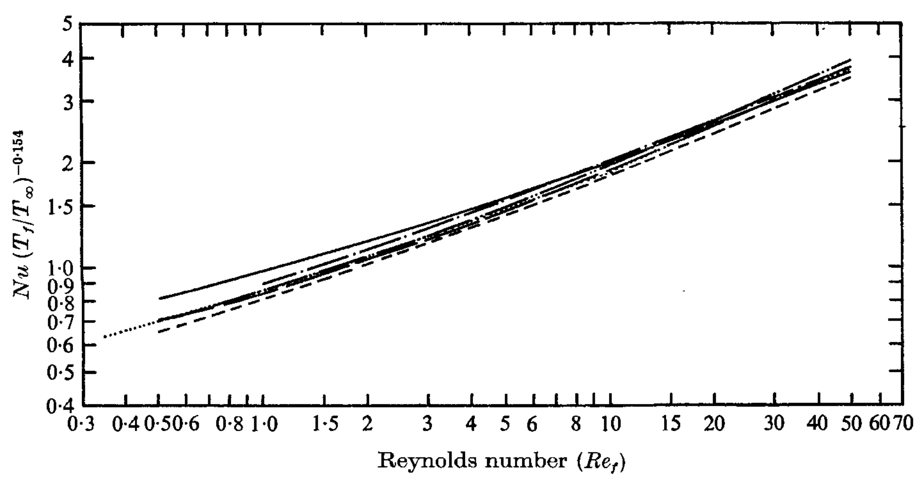

FIGURE 2. Comparison of proposed correlation with existing correlations for forced convection. ——, proposed correlation; —· ·—, van der Hegge Zijnen; - - - -, Collis & Williams; —·—, Hilpert; — —, Tsubouchi & Masuda; · · · · · ·, McAdams.

6.2. *Natural convection*

An equation of the following form is proposed for the natural convection régime

$$Nu\,[T_f/T_\infty]^{-m} = A + B\,Ra^n. \tag{14}$$

This is different from the normal type of correlation in that it includes a temperature loading factor. The temperature loading factor index m used was 0·154, the same as that used for the forced convection case. The experimental points are shown in figure 3 (*a*) plotted as $Nu\,[T_f/T_\infty]^{-0\cdot154}$ against Ra. It is evident from the graph that the use of a temperature loading factor is acceptable as it does not produce any temperature-dependent deviation which would be evident between the maximum temperature points of the 0·032 in. cylinder and the minimum temperature points of the 0·0495 in. cylinder.

A computer solution was carried out to find the constants A, B and n which gave the equation

$$Nu\,[T_f/T_\infty]^{-0\cdot154} = 0\cdot525 + 0\cdot422 Ra^{0\cdot315}. \tag{15}$$

This line is indicated on figure 3 (*a*).

In order to use a vectorial addition for mixed convection it is convenient that the constant A be the same for both natural and forced convection.

Another correlation was therefore attempted by fixing A at the same value as that found for forced convection, i.e. $A = 0\cdot384$, which gave

$$Nu\,[T_f/T_\infty]^{-0\cdot154} = 0\cdot384 + 0\cdot59 Ra^{0\cdot184}. \tag{16}$$

This is also plotted in figure 3 (*a*) and it can be seen that it satisfies the experimental points reasonably well.

Both equations (15) and (16) are shown on figure 3 (*b*), together with the correlations referred to in table 1. The modified correlation, equation (16), compares well with those of the other workers and in particular with that of van der Hegge Zijnen (1956).

6.3. *Mixed convection*

The forced and natural convection correlations, being of the same form, now enable the equivalence previously suggested as

$$Re \equiv (2Gr)^{0\cdot5} = 1\cdot687 Ra^{0\cdot5} \quad \text{(for air)}$$

to be replaced by a more realistic approximation. The same Nusselt number is obtained from both forced and natural correlations when

$$Re_f = 1\cdot03 Ra^{0\cdot418}.$$

Substituting this value in the vector addition gives the general correlation. The modified Re_{eff} equation is now

$$Re_{\text{eff}}^2 = Re_f^2\left[1 + 2\cdot06\left[\frac{Ra^{0\cdot418}}{Re_f}\right]\cos\theta + 1\cdot06\left[\frac{Ra^{0\cdot836}}{Re_f^2}\right]\right]. \tag{17}$$

Figures 4 (*a*)–(*c*) show all the experimental data, except those for pure natural convection, plotted without any correction being attempted, i.e. assuming it to be a forced convection process alone and they are compared with the forced convection correlation. For parallel flow, figure 4 (*a*), all of the experimental data lie above the forced convection line. This is as would be expected as the natural convection is aiding the forced convection. For cross-flow, figure 4 (*b*), some of the experimental points fall below the forced convection line whilst most of the points remain above. This is probably due to the natural convection disturbing the flow pattern at certain points and reducing the heat-transfer coefficient slightly. For contra flow, figure 4 (*c*), approximately half the experimental points are above and the other half below. To assist in the understanding of this graph two lines have been drawn connecting the maximum *Ra* values of the 0·032 in. and 0·0495 in. cylinders. From these it will be seen that as the Reynolds number is reduced, a point is reached when the natural convection opposes the forcing convection and the heat-transfer coefficient decreases. This continues until the influence of natural convection becomes greater than that of the forced and the heat-transfer coefficient then increases up to a maximum which is the natural convection value. Hence in this region a measurement of Nusselt number, which is essentially that which is made in an anemometer, would not yield a unique value of Reynolds number.

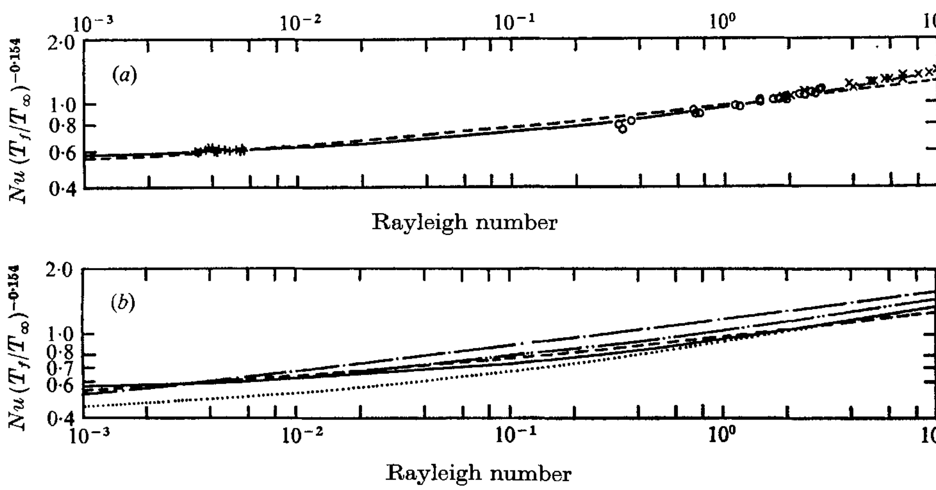

FIGURE 3. (*a*) Proposed natural convection correlation:
——, computer curve fit $Nu\,(T_f/T_\infty)^{-0.154} = 0.525 + 0.422 Ra^{0.315}$; - - -, suggested curve fit $Nu\,(T_f/T_\infty)^{-0.154} = 0.384 + 0.59 Ra^{0.184}$; +, $d = 0.004$ in.; ○, $d = 0.032$ in.; ×, $d = 0.0495$ in.
(*b*) Comparison of proposed correlation with existing correlations for natural convection.
—·—, Mikheyev; —· ·—, van der Hegge Zijnen; ——, computer curve fit; - - - -, suggested curve fit; ......, Tsubouchi & Masuda.

The correlation, equation (17), was applied to all the data including the natural convection points and all the results were plotted for each direction individually on figure 5 (*a*)–(*c*). The solid line on each graph is the forced convection correlation which is also the correlation for mixed convection obtained by substituting Re_{eff} for Re_f.

Considering figure 5 (*a*), all the data for the parallel flow lie within ± 10 % of the correlating line.

For cross-flow, figure 5 (*b*), a certain amount of the data lies away from the correlating line but generally the form is correct.

For contra flow, figure 5 (*c*), a large amount of the data lies away from the line, but this is to be expected as with the type of correlation suggested it is possible to obtain Re_{eff} equal to zero and hence the correlation must fail.

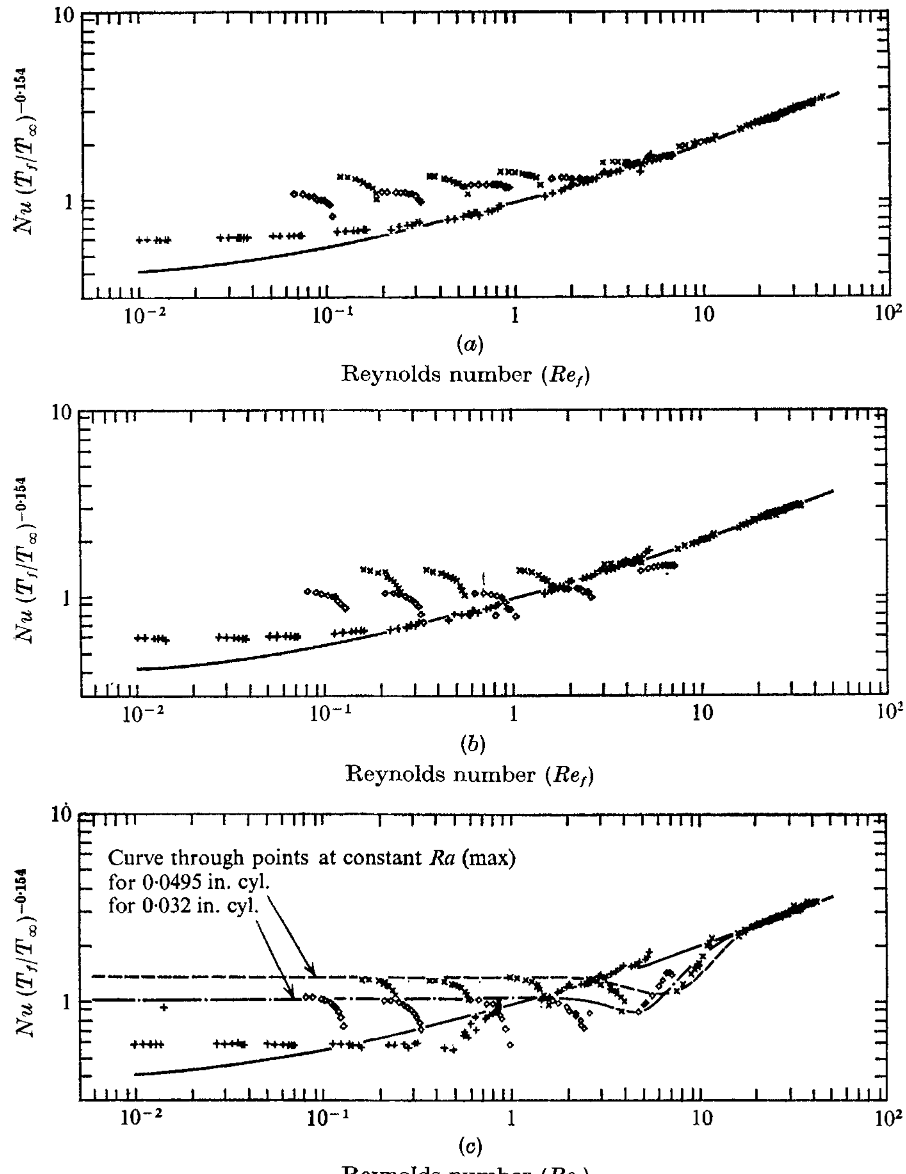

FIGURE 4. Proposed forced convection correlation:

$$Nu(T_f/T_\infty)^{-0.154} = 0.384 + 0.581 Re_f^{0.439}$$

applied to the mixed flow régime. $+$, $d = 0.004$ in.; $\diamond$, $d = 0.032$ in.; $\times$, $d = 0.0495$ in. (*a*) parallel flow; (*b*) cross-flow; (*c*) contra flow.

FIGURE 5. Proposed correlation applied to the mixed flow régime. Correlation:

$$Nu\,(T_f/T_\infty)^{-0.154} = 0.384 + 0.581 Re_{\text{eff}}^{0.439};$$

$$Re_{\text{eff}} = Re_f^2[1 + 2.06 \cos\theta\, Ra^{0.418}/Re_f + 1.06 Ra^{0.836}/Re_f^2].$$

$+$, $d = 0.004$ in.; $\diamond$, $d = 0.032$ in.; $\times$, $d = 0.0495$ in. (*a*) parallel flow; (*b*) cross-flow; (*c*) contra flow. (*d*) Contra flow with experimental data in the range $0.25 < Ra^{0.418}/Re_f < 2.5$ omitted.

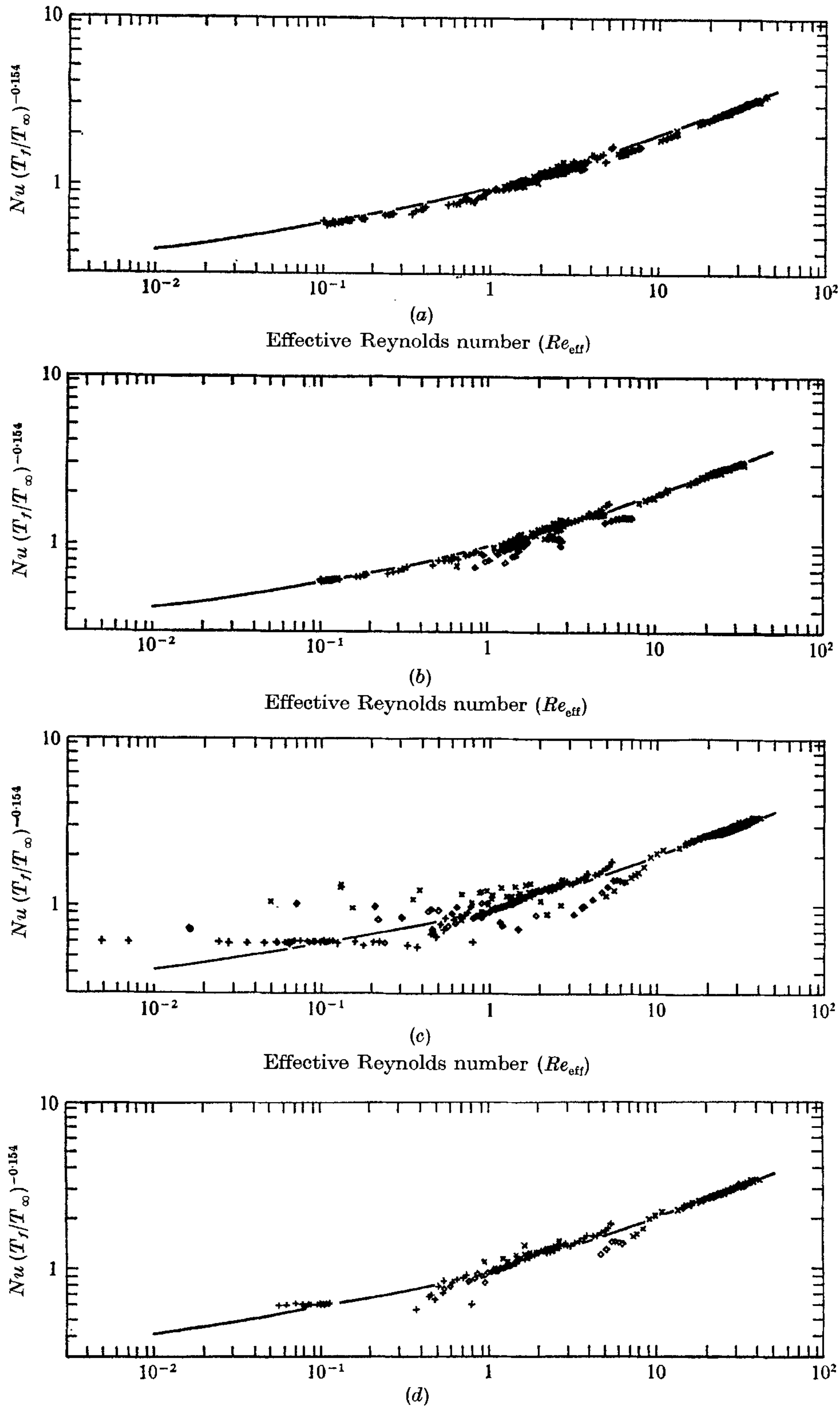

FIGURE 5. For legend see facing page.

$$Re_{\text{eff}} = 0 \quad \text{when} \quad \frac{Ra^{0\cdot418}}{Re_f} = 1.$$

The correlation cannot be used in contra flow when

$$0\cdot25 > \frac{Ra^{0\cdot418}}{Re_f} > 2\cdot5. \tag{18}$$

If it is used in this range unacceptable errors > 10 % will result. Figure 5(*d*) is a repeat of figure 5(*c*) with all the points in this range omitted and it shows a fairly satisfactory result.

The correlation is useful for determining the values at which natural convection takes effect on the forcing flow. It should be noted that the effect of natural convection on forced convection will take place at different values of $Ra^{0\cdot418}/Re_f$, dependent upon θ.

7. Conclusions

(1) The purely forced convection experimental data agree best with the correlation of Hilpert (1933). (2) The purely natural convection experimental data compare well with the van der Hegge Zijnen (1956) correlation. (3) The results reported in this work indicate it would be difficult to construct a low-speed hot-wire anemometer. In the contra flow régime particularly, it would not be possible from the wire voltage to infer the Reynolds number. For the other régimes it might be possible providing the direction of the air flow was known. (4) Hot-wire anemometers may be used down to the following conditions before natural convection has an effect of more than 10 % on the indicated Reynolds number which would be inferred by assuming purely forced flow.

Parallel flow	$Re_f = 10Ra^{0\cdot418}$
Cross flow	$Re_f = 2\cdot2Ra^{0\cdot418}$
Contra flow	$Re_f = 9Ra^{0\cdot418}$

When Re_f is lower than these limits the errors increase. (5) The empirical correlation given may be used with confidence in range

$$10^{-2} > Re_f > 40 \text{ and } 10^{-3} > Ra > 10.$$

In the contra flow situation the correlation is not valid when

$$0\cdot25 > \frac{Ra^{0\cdot418}}{Re_f} > 2\cdot5.$$

When the value of $Ra^{0\cdot418}/Re_f$ equals either of these two limits the error in the Nusselt number predicted is ± 10 % and this increases to an unacceptable level between the limits previously mentioned.

The correlation is

$$Nu\,[T_f/T_\infty]^{-0\cdot154} = 0\cdot384 + 0\cdot581 Re_{\text{eff}}^{0\cdot439},$$

where

$$Re_{\text{eff}}^2 = Re_f^2\left[1 + 2\cdot06\frac{Ra^{0\cdot418}}{Re_f}\cos\theta + 1\cdot06\frac{Ra^{0\cdot836}}{Re_f^2}\right].$$

Some useful initial development of the apparatus was carried out by M. J. Pomfret.

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